1. **y** is inversely proportional to **x**.
   \[ y = \frac{k}{x} \]

   *y* = 16 when **x** = \( \frac{1}{2} \)

   Write an expression for **y** in terms of **x**.

   \[
   16 = \frac{k}{0.5} \\
   k = 16 \times 0.5 \\
   k = 8 \\
   y = \frac{8}{x}
   \]

2. A pebble is thrown vertically upwards.
   It has an initial speed of \( u \) metres per second.
   The pebble reaches a maximum height of \( h \) metres, before falling vertically downwards.
   It is known that \( h \) is directly proportional to \( u^2 \).

   When the pebble is thrown with an initial speed of 10m/s it reaches a maximum height of 5m.

   (a) Calculate the maximum height reached when the pebble is thrown with an initial speed of 12 m/s.

   \[
   h = ku^2 \\
   u = 10 \text{ m/s} \\
   h = 5 \text{ m} \\
   S = k10^2 \\
   k = \frac{5}{100} = 0.05
   \]

   \[
   \therefore h = 0.05u^2 \\
   \text{when } u = 12 \text{ m/s} \\
   h = 0.05 \times 12^2 \\
   h = 7.2 \text{ m}
   \]

   (b) Find the initial speed of the pebble if the maximum height reached is 16 m.

   \[
   h = 0.05u^2 \\
   h = 16 \\
   16 = 0.05u^2 \\
   u^2 = \frac{16}{0.05} \\
   u = \sqrt{320} = 8.4848 \\
   = 17.89 \text{ m/s (2dp)}
   \]
3. Sketch a graph on the axes below that shows that $y$ is directly proportional to $x$.

![Graph showing direct proportion]

4. At a constant temperature, the volume of a gas $V$ is inversely proportional to its pressure $p$.

By what percentage will the pressure of a gas change if its volume increases by 25%?

$$V \propto \frac{1}{p} \quad V = \frac{k}{p} \quad P = \frac{k}{V} \quad P_2 = \frac{k}{1.25V}$$

$$\left[\left(\frac{k}{1.25V} - \frac{k}{V}\right) \div \frac{k}{V}\right] \times 100 = \left(\frac{k}{1.25V} - \frac{k}{V}\right) \times V \times \frac{100}{kV} = \frac{V}{1.25V} - \frac{k}{kV} \times 100$$

$$\Rightarrow \left(\frac{1}{1.25} - 1\right) \times 100 \quad \text{.........................} 20 \quad \% \quad [4]$$

5. $d$ is inversely proportional to $c$

When $c = 280$, $d = 25$

Find the value of $d$ when $c = 350$

$$d \propto \frac{1}{c} \quad d = \frac{k}{c}$$

$$25 = \frac{k}{280}$$

$$25 \times 280 = k \quad \Rightarrow \quad k = 7000$$

$$\therefore \quad d = \frac{7000}{c}$$

when $c = 350$

$$d = \frac{7000}{350} = 20$$

[3]
6. \( y \) is directly proportional to \( \sqrt{x} \)

\[
y \propto \sqrt{x}
\]

\[
y = k \sqrt{x}
\]

\[
2 = k \sqrt{36}
\]

\[
k = \frac{2}{\sqrt{36}} = \frac{2}{6} = \frac{1}{3}
\]

Work out the value of \( a \).

when \( y = S \)

\[
S = \frac{1}{3} \sqrt{a} \\
1S = \sqrt{a} \quad (\text{sq}) \\
a = 1S^2 \\
= 225
\]

7. \( y \) is directly proportional to \( x \).

Which graph shows this?

Circle the correct letter.
8. a) \( y \) is directly proportional to \( \sqrt{x} \)

\( y \) is 75 when \( x = 100 \).

Find a formula linking \( x \) and \( y \).

\[
\begin{align*}
y &\propto \sqrt{x} \\
y &= k\sqrt{x} \\
75 &= k\sqrt{100} \\
k &= \frac{75}{10} = 7.5
\end{align*}
\]

\[
y = 7.5\sqrt{x}
\]

b) \( y \) is inversely proportional to \( x^2 \) and \( y = 3 \) when \( x = 12 \).

Show that \( y = 27 \) when \( x = 4 \).

\[
\begin{align*}
y &\propto \frac{k}{x^2} \\
y &= \frac{k}{x^2} \\
3 &= \frac{k}{12^2} \\
k &= 3 \times 144 \\
k &= 432
\end{align*}
\]

\[
y = \frac{432}{x^2} \\
\text{when } x = 4 \\
y = \frac{432}{16} = 27
\]

9. At a depth of \( x \) metres, the temperature of the water in an ocean is \( T \) °C.

At depths below 900 metres, \( T \) is inversely proportional to \( x \).

\( T \) is given by

\[
T = \frac{4500}{x}
\]

a) Work out the difference in the temperature of the water at a depth of 1200 metres and the temperature of the water at a depth of 2500 metres.

\[
\begin{align*}
T &= \frac{4500}{1200} \\
&= 3.75 \degree C \\
T &= \frac{4500}{2500} \\
&= 1.8 \degree C
\end{align*}
\]

\[\text{Difference} = 3.75 - 1.8 = 1.95 \degree C\]
Here are four graphs.

One of the graphs could show that T is inversely proportional to x.

b) Write down the letter of this graph.

....................................................... [1]

10 y is inversely proportional to x

When x = 1.5, y = 36

Find the value of y when x = 6

\[
y = \frac{k}{x} \quad y = \frac{k}{x}
\]

\[
36 = \frac{k}{1.5}
\]

\[
k = 36 \times 1.5
\]

\[
k = 54
\]

\[
y = \frac{54}{6} = 9
\]

when x = 6  \quad y = \frac{54}{6} = 9

[3]

11. D is directly proportional to the cube of n.

\[
D \propto n^3 \quad D = kn^3
\]

Mary says that when n is doubled, the value of D is multiplied by 6

Mary is wrong. \[D_1 = kn^3 \quad D_2 = k(2n)^3\]

the value is multiplied by 8, i.e. \[2^3\]

[1]
12. These graphs show four different proportionality relationships between $y$ and $x$.

Match each graph with a statement in the table below.

<table>
<thead>
<tr>
<th>Proportionality relationship</th>
<th>Graph letter</th>
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<tbody>
<tr>
<td>$y$ is directly proportional to $x$</td>
<td>D</td>
</tr>
<tr>
<td>$y$ is inversely proportional to $x$</td>
<td>A</td>
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<tr>
<td>$y$ is proportional to the square of $x$</td>
<td>B</td>
</tr>
<tr>
<td>$y$ is inversely proportional to the square of $x$</td>
<td>C</td>
</tr>
</tbody>
</table>
13. A pendulum of length \( L \) cm has time period \( T \) seconds.

\[ T \text{ is directly proportional to the square root of } L. \quad L \propto \sqrt{L} \quad T = k\sqrt{L} \]

The length of the pendulum is increased by 40%.

Work out the percentage increase in the time period.

\[ T_1 = k\sqrt{L} \quad T_2 = k\sqrt{1.4L} \]

\[ \frac{k\sqrt{1.4L} - k\sqrt{L}}{k\sqrt{L}} = \frac{k\sqrt{1.4L}}{k\sqrt{L}} - \frac{k\sqrt{L}}{k\sqrt{L}} \]

\[ = \frac{1.4 - 1}{1} \times 100 \]

\[ = 18.32\% \] [3]

14. \( w \) is directly proportional to \( y \)

\( w \) is inversely proportional to \( x^2 \)

\[ w \propto y \quad w = ky \]

a) When \( y = 4 \), \( w = 14 \)

Work out the value of \( w \) when \( y = 9 \)

\[ w = ky \]

\[ 14 = k \times 4 \]

\[ k = \frac{14}{4} = 3.5 \]

\[ w = 3.5y \]

When \( y = 9 \),

\[ w = 3.5 \times 9 \]

\[ = 31.5 \] [2]

b) When \( x = 2 \), \( w = 5 \)

Work out the value of \( w \) when \( x = 10 \)

\[ w \propto \frac{1}{x^2} \]

\[ 5 = \frac{k}{2^2} \]

\[ k = 20 \]

\[ w = \frac{20}{x^2} \]

When \( x = 10 \),

\[ w = \frac{20}{10^2} = \frac{20}{100} \]

\[ = 0.2 \] [3]

c) Which graph shows the relationship between \( y \) and \( x \)?

Circle the correct letter.

\[ w \propto y \quad w \propto \frac{1}{x^2} \quad w = \frac{20}{x^2} \]

\[ w = 3.5y \]
15. \( y \) is directly proportional to \( x \) and \( k \) is a constant.
Circle the correct equation.

- \( y = x + k \)
- \( y = kx \)
- \( y = \frac{k}{x} \)
- \( y = x - k \)

16. The time of each swing of a pendulum, length \( l \) cm, is \( T \) seconds.

\( T \) is directly proportional to the square root of \( l \).

When \( l = 64 \) \( T = 1.6 \)

Work out the value of \( T \) when \( l = 132.25 \)

\[
T = 0.2\sqrt{l}
\]

\[
T = 0.2 \times \sqrt{132.25} = 2.3
\]
CREDITS AND NOTES

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Notes:

These questions have been retyped from the original sample/specimen assessment materials and whilst every effort has been made to ensure there are no errors, any that do appear are mine and not the exam board’s (similarly any errors I have corrected from the originals are also my corrections and not theirs!).

Please also note that the layout in terms of fonts, answer lines and space given to each question does not reflect the actual papers to save space.

These questions have been collated by me as the basis for a GCSE working party set up by the GLOW maths hub - if you want to get involved please get in touch. The objective is to provide support to fellow teachers and to give you a flavour of how different topics “could” be examined. They should not be used to form a decision as to which board to use. There is no guarantee that a topic will or won’t appear in the “live” papers from a specific exam board or that examination of a topic will be as shown in these questions.

Links:

AQA http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300
OCR http://ocr.org.uk/gcsemaths
WJEC Eduqas http://www.eduqas.co.uk/qualifications/mathematics/gcse/

Contents:

This version contains questions from:

AQA – Sample Assessment Material, Practice set 1 and Practice set 2
OCR – Sample Assessment Material and Practice set 1
Pearson Edexcel – Sample Assessment Material, Specimen set 1 and Specimen set 2
WJEC Eduqas – Sample Assessment Material